Turbulence Models for Wall Boundary Layers

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Abstract

BOUNDARY-layer equations with boundary conditions corresponding to a wide range of plane, two-dimensional flows, including the 14 mandatory test cases of the 1968 AFOSR-IFD-Stanford Conference and those over the sub- and supercritical airfoils of Nakayama, have been solved. Results were obtained with turbulence models based on an algebraic eddy-viscosity formula, the equations for turbulent kinetic energy and dissipation rate, and equations for the Reynolds stresses and dissipation rate. They are shown to depend upon the model in an almost random manner. The cost penalty associated with the higher order model is quantified and it is concluded that, for the flows considered, a well-founded algebraic eddy-viscosity model is preferred.

Contents

In recent papers, Cebeci and Meier[1] and Cebeci and Chang[2] have considered the application of turbulence models to two- and three-dimensional boundary layers. In the first paper, two eddy-viscosity models and a Reynolds stress model were used in the calculation of two of the boundary-layer flows documented by Coles and Hirst,[3] and an algebraic eddy-viscosity model was used for a small number of two- and three-dimensional flows. In the second paper, two eddy-viscosity models were employed in the calculation of the infinite swept-wing flow of van den Berg and Elsenaar[4] and in the three-dimensional flow of East and Hoxey.[5] In both papers the models were evaluated in terms of the agreement between the measured and calculated results with the general conclusion that a good algebraic specification of eddy viscosity is preferred. However, this conclusion was limited to the narrow range of flows considered.

The present synoptic, based on Ref. 6, extends the results of Cebeci and Meier[1] in two ways. It considers the application of the same two eddy-viscosity models, i.e., the algebraic formulation of Cebeci and Smith[7] and the k-ϕ model of Hanjalic and Launder,[8] to a much wider range of the data assembled in Ref. 3 and to the flow near the trailing edge of a subcritical airfoil at zero angle of attack and of a supercritical airfoil at 4-deg angle of attack. The use of a Reynolds stress model is also considered; however, in contrast to the earlier calculations, the Reynolds-stress transport approach of Launder et al.[9] is preferred to that of Bradshaw et al.[10] and is applied in a form incorporating wall functions to the supercritical airfoil flow to examine its ability to represent a flow subjected to a strong adverse pressure gradient and also to allow comparison of the three types of models on a basis of cost, as well as accuracy. The criteria for choice of a turbulence model inevitably includes the ability of the model to represent aspects of a range of flow configurations but must also include consideration of the computational resources the model requires. These resources are considered here within the framework of the numerical procedure of Keller,[11] which is readily documented in terms of the equations to be solved, has been widely used, and is known to be numerically accurate and efficient.

The results of the calculations are presented in 25 figures, two of which are reproduced herein—Figs. 1 and 2—with one table—Table 1. The first 24 figures show the variations of local skin-friction coefficient, shape factor, and momentum thicknesses obtained with the two eddy-viscosity turbulence models, and the last figure which corresponds to Nakayama's[12] supercritical airfoil at 4-deg angle of attack shows results obtained with the three models. The table allows comparison of corresponding computer storage and cost. The data selected include 14 of the 16 flows assigned as mandatory test cases at the AFOSR-IFD-Stanford Conference,[3] with two flows omitted on the grounds that they involve axial symmetry. In the selection of remaining flows, only plane flows in the absence of temperature gradients have been considered. The measurements of Nakayama[12] have been considered because they were obtained on airfoils of immediate practical relevance and appear to be of high quality.

As discussed by Cebeci and Meier,[1] boundary and initial conditions can influence the calculated results and, for the results presented, the initial velocity profile and wall shear stress, together with the freestream velocity distribution, were identical for the calculation with each model. Similarly, the initial distributions of normal stresses in the Reynolds stress transport model were compatible with the turbulent kinetic energy of the two-equation model and the initial shear-stress distributions were compatible with the initial assumptions for length scale and dissipation rate. It follows, of course, that the calculations of each figure began with the same origin.

It is evident that the discrepancies between calculations and experiments increase as separation approaches but this is undoubtedly due in part to the increasing influence of cross-stream pressure gradient. In general, the three models yield similar results except perhaps in two cases where discrepancies

Table 1 Computer resources required for the three turbulence models of Fig. 2

	Algebraic eddy-viscosity model	Two-equation model	Stress-equation model
Computer storage, Kbytes	96	124	332
Relative CPU time	1	4	30

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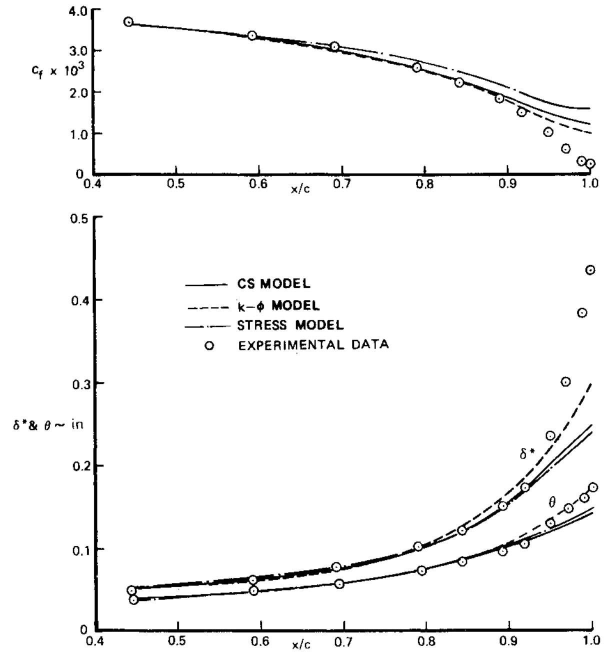

Fig. 1 Supercritical airfoil of Ref. 12, $\alpha = 4$; upper surface.

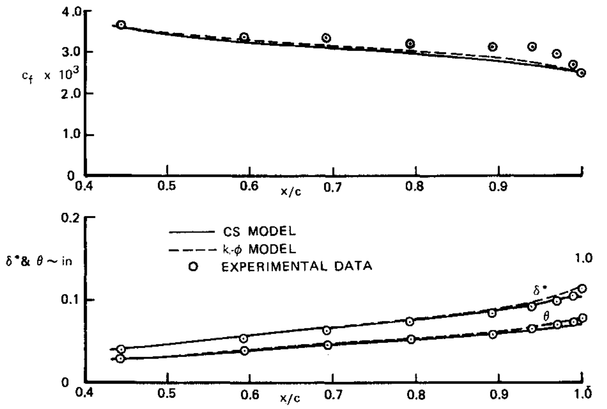

Fig. 2 Subcritical airfoil of Ref. 12, $\alpha = 0$; lower surface.

are consistent with those previously discussed by Nituch et al.,[13] who suggested an improved form of the algebraic eddy-viscosity model to represent relaxing flows and equilibrium layers in strong adverse pressure gradients.

The results for Nakayama's supercritical airfoil at 4-deg angle of attack have some similarities with those obtained in relation to Bradshaw's adverse pressure-gradient flow,[14] in that the integral parameters are well represented up to a shape factor of around 2. They are shown in Fig. 1 and may be contrasted with similar results for a subcritical airfoil at zero angle of attack in Fig. 1. The skin-friction coefficients are also better represented up to this value, which corresponds to x/c of around 0.9, although the stress model gives noticeably worse results. In the range of x/c from 0.9 to 1, none of the models is adequate, but normal stresses and normal pressure gradients may account for part of the problem. It is interesting to note that all three models were able to represent the flow over the airfoil at 4-deg angle of attack although, once again, the stress model gave the poorest representation of skin-friction coefficient with discrepancies up to 25% above the measurements.

It has been shown that the required computer storage and CPU time increase substantially with the complexity of the turbulence model (see Table 1), so that the calculation of the airfoil flow with a Reynolds-stress equation model is 30 times more expensive than with an algebraic eddy-viscosity model. An efficient computer program, such as that used here, ensures that the most expensive of the calculations corresponds to around 45 s of computer time and, compared with many engineering calculations, this may seem small. The application of boundary-layer calculation methods to aid design is, however, complex in that it is likely to form part of an overall calculation which involves the solution of potential-flow equations. Experience with interactive methods for two-dimensional flows shows that at least five iterations may be required to achieve convergence. In addition, it is usually necessary to perform calculations for a range of angles of attack and configurations so that the added cost associated with the higher order turbulence models becomes increasingly important. The added costs associated with the calculation of three-dimensional flows are even greater.

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References

[1] Cebeci, T. and Meier, H.U., "Modelling Requirements for the Calculation of the Turbulent Flow Around Airfoils, Wings and Bodies of Revolution," AGARD CP 271, Paper 16, 1979.

[2] Cebeci, T. and Chang, K.C., "On the Turbulence-Modelling Requirements of Three-Dimensional Boundary-Layer Flows," *Recent Contributions to Fluid Mechanics*, edited by W. Haase, Springer-Verlag, Berlin, Heidelberg, 1982, p. 31.

[3] Coles, D. and Hirst, E.A., "Computation of Turbulent Boundary Layers—1968," *AFOSR-IFD-Stanford Conference, Vol. 2*, Thermosciences Div., Stanford University, Stanford, CA, 1969.

[4] van den Berg, B. and Elsenaar, A., "Measurements in a Three-Dimensional Turbulent Boundary Layer in an Adverse Pressure Gradient Under Infinite Swept-Wing Conditions," NLR TR 72092U, 1972.

[5] East, L.F. and Hoxey, R.P., "Low Speed Three-Dimensional Turbulent Boundary-Layer Data," RAE TR 69041, 1969.

[6] Cebeci, T., Chang, K.C., Li, C., and Whitelaw, J.H., "Turbulence Models for Wall Boundary Layers," Mechanical Engineering Dept., California State Univ., Long Beach, Rept. ME-84-2, 1984.

[7] Cebeci, T. and Smith, A.M.O., *Analysis of Turbulent Boundary Layers*, Academic Press, New York, 1974.

[8] Hanjalic, W.P. and Launder, B.E., "Sensitizing the Dissipation Equation to Irrotational Strains," *Journal of Fluids Engineering, Transactions of ASME*, Vol. 102, March, 1980, pp. 34-40.

[9] Launder, B.E., Reece, G., and Rodi, W., "Progress in the Development of a Reynolds-Stress Turbulence Closure," *Journal of Fluid Mechanics*, Vol. 68, April 1975, p. 537.

[10] Bradshaw, P., Ferriss, D.H., and Atwell, W.P., "Calculation of Boundary-Layer Development Using the Turbulent Energy Equations," *Journal of Fluid Mechanics*, Vol. 28, May 1967, p. 593.

[11] Keller, H.B., "A New Difference Scheme for Parabolic Problems," *Numerical Solutions of Partial-Differential Equations*, Vol. II, edited by J. Bramble, Academic Press, New York, 1970.

[12] Nakayama, A., "Measurements of Attached and Separated Turbulent Flows in the Trailing-Edge Regions of Airfoils," *Numerical and Physical Aspects of Aerodynamic Flows II*, edited by T. Cebeci, Springer-Verlag, New York, 1984, p. 253.

[13] Nituch, M.J., Sjolander, S., and Head, M.R., "An Improved Version of the Cebeci-Smith Eddy-Viscosity Model," *Aeronautical Quarterly*, Vol. 29, Aug. 1978, p. 207.

[14] Bradshaw, P., "The Response of a Constant Pressure Boundary Layer to the Sudden Application of an Adverse Pressure Gradient," NPL Aero Rept. 1219, 1967.